1. Use the given information to test the following hypothesis.

\[ H_0: \mu = 28, \quad \bar{X} = 16, \quad S = 4, \quad n = 16, \quad \alpha = 0.01 \]

\[ H_a: \mu \neq 28 \]

\[ t* = \frac{16 - 28}{4} = -12 \]

\[ \Rightarrow \mu \neq 28 \]

Explain.

**P-Value Method**

\[ P = 2 \cdot P( t > 12 ) \]

\[ = 2 \cdot \text{cdf}(12, 1000, 15) \]

\[ = 2 \cdot (2.1613 \times 10^{-9}) \]

\[ = 4.3227 \times 10^{-9} \]

\[ \Rightarrow P \approx 0.001 \]

\[ \alpha = 0.005 \]

**Table**
2. Mr. Clean claims that at least 60% of the female car owners have a hybrid car. In a random sample of 100 female car owners, 58 owned a hybrid car. Is there enough evidence to support Mr. Clean’s claim?

\[ H_0: p = 0.6 \]
\[ H_a: p < 0.6 \]

\[ \hat{p} = \frac{58}{100} = 0.58 \]

\[ z^* = \frac{0.58 - 0.60}{\sqrt{0.6(0.4)/100}} = -0.41 \]

\[ P = P(z < z^*) = P(z < -0.41) = \text{normalcdf}(-1000, -0.41) \approx 0.34 \]

\[ \hat{p} = 0.58 > \alpha = 0.05 \]
3.- A Special Forces Military Recruiter claims that the height of his recruits have a standard deviation of 5 inches. A random sample of 20 of his recruits had standard deviation of 7 inches. At $\alpha = 0.01$, is there enough evidence to support his claim?

\[ n=20, \ s=7, \ \alpha=0.01 \]

51) $H_0: \ \sigma = 5$

52) $\chi^2* = \frac{(n-1)s^2}{\sigma^2} = \frac{(20)(7)^2}{5^2} = 39.2$

53) $\chi^2_R = \chi^2_{0.005, 23} = 44.181$

$\chi^2_L = \chi^2_{0.995, 23} = 9.260$

54) Fail to reject $H_0$ (Accept $H_0$)

$\Rightarrow \ \sigma = 5$

$\Rightarrow$ There is enough evidence to support the recruiter's claim that the standard deviation is 5 inches.

P-Value Method

53) $P = 2P(\chi^2 > \chi^2*)$

$= 2P(\chi^2 > 39.2)$

$= 2 \cdot \chi^2_{cdf}(39.2, 1000, 20)$

54) $P = 0.4 > \alpha = 0.01$

$\Rightarrow$ Accept $H_0$

(Fail to reject)
4.- Use the given information to test the following hypothesis.

\[ H_0 : p_1 = p_2, \quad n_1 = 400, \quad x_1 = 200, \quad n_2 = 500, \quad \hat{p}_2 = 0.7 \]

\[ H_a : p_1 \neq p_2 \]

\[ \Delta = 0.05 \]

\[ \bar{p}_1 = \frac{200}{400} = 0.5 \]

\[ \bar{p}_2 = \frac{200 + 350}{400 + 500} = \frac{550}{900} = 0.61 \]

\[ \bar{p} = \frac{200 + 350}{400 + 500} = 0.61 \]

\[ z = \frac{0.7 - 0.5}{\sqrt{(0.61)(0.39)(\frac{200}{400} + \frac{1}{500})}} \approx -6.12 \]

\[ 2 \cdot 0.025 = \pm 1.96 \]

\[ z = 6.12 \]

\[ \Rightarrow \text{Reject } H_0 \]

\[ \Rightarrow p_1 \neq p_2 \]

\[ \Rightarrow \text{There is a difference} \]
5.- Apple claims that the battery life of the iPhone is better than that of the Google phone. A sample is taken from each company. The Apple sample \( (n = 20) \) had a mean of 12 hours and standard deviation of 3 hours, while the Google sample \( (n = 24) \) had a mean of 10 hours with a standard deviation of 4 hours. Is there a difference in the battery life of the two companies?

\[
\overline{X}_1 = 12 \quad S_1 = 3 \quad n_1 = 20 \\
\overline{X}_2 = 10 \quad S_2 = 4 \quad n_2 = 24
\]

\( S1) \quad H_0: \mu_1 = \mu_2 \quad (\text{no difference in battery life}) \\
H_a: \mu_1 \neq \mu_2

\( S2) \quad t^* = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \approx 1.89

\( S3) \quad P \text{-value method} \\
P = 2 \cdot P(t > 1.89) \approx 0.07

\( S4) \quad P = 0.07 > \alpha = 0.05 \\
\Rightarrow \text{Accept } H_0
6.- Consider the following data.

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>y</th>
<th>$x^2$</th>
<th>$y^2$</th>
<th>$xy$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>1</td>
<td>16</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>1</td>
<td>36</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>4</td>
<td>36</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>9</td>
<td>64</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td>49</td>
<td>100</td>
<td>70</td>
<td></td>
</tr>
<tr>
<td>Σ</td>
<td>14</td>
<td>34</td>
<td>64</td>
<td>252</td>
<td>116</td>
</tr>
</tbody>
</table>

$S_{xx} = 24.8$, $S_{yy} = 20.8$, $S_{xy} = 19.2$

(a) Find $r$

$$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{19.2}{\sqrt{(24.8)(20.8)}} = 0.845$$

(b) Find $\hat{y}$

$$m = \frac{S_{xy}}{S_{xx}} = \frac{19.2}{24.8} = 0.77$$

$$b = \bar{y} - m\bar{x} = 6.8 - (0.77)(2.8) = 4.6$$

(c) Predict $y$ when $x$ is 4

$$\hat{y} = 0.77(4) + 4.6 = 7.7$$

(d) Test the hypothesis $\rho = 0$ vs $\rho \neq 0$, with $\alpha = 0.10$

$S_1$) $H_0: \rho = 0$
$S_2$) $t^* = 0.85\sqrt{\frac{3}{1-(0.85)^2}} \approx 2.794$

$S_3$) $\mu_{0.05, 3} = \pm 3.182$
7.- Use the given information to test the following hypothesis.

\[ H_0 : \sigma_1 = \sigma_2 \quad \quad S_1 = 15, \quad S_2 = 20, \quad n_1 = 9, \quad n_2 = 7 \]

\[ H_a : \sigma_1 \neq \sigma_2 \]

51) \[ F* = \frac{20^2}{15^2} = 1.78 \]

d.f. \( N = 6 \)

d.f. D = 8

52) \[ \text{EE: } F = F_{0.025, 6, 8} = 4.65 \]

53) \[ \text{EE: } F = F_{0.025, 6, 8} = 4.65 \]

54) Fail to reject \( H_0 \)

\[ \Rightarrow \sigma_1 = \sigma_2 \Rightarrow \text{no difference} \]

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**P-Value Method**

51) 52) Same

53) \[ P = 2 \cdot P(F^2 > 1.78) \]

\[ = 2 \cdot F_{cdf}(1.78, 1000, 6, 8) \]

\[ = 0.44 \]

54) \[ P = 0.44 > \alpha = 0.05 \]

\[ \Rightarrow \text{Fail to reject } H_0. \]
8.- According to a recent survey a high percentage of incoming college students place in developmental mathematics because they did not prepare for the placement test. A group of 5 students that tested in developmental mathematics was given a two week preparation during the summer. At the end of the two weeks the students retook the test. The following table summarizes the results. Is there a difference in the scores after taking the two week preparation. Use $\alpha = 0.05$ (10 pts)

<table>
<thead>
<tr>
<th>$x_1$</th>
<th>60</th>
<th>50</th>
<th>40</th>
<th>55</th>
<th>62</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_2$</td>
<td>90</td>
<td>85</td>
<td>81</td>
<td>58</td>
<td>69</td>
</tr>
</tbody>
</table>

\[ n = 5 \]

\[ \bar{d} = 21.8, \quad S_d = 19.3 \]

5(i) $H_0: \mu_d = 0$

5(ii) $H_a: \mu_d \neq 0$

5(iii) $t* = \sqrt{n} \bar{d} \over S_d = \sqrt{5} \left( \frac{21.8}{19.3} \right) \approx 2.48$

5(iv) REJECT $H_0$: $t* = 2.48$

5(v) Fail to reject $H_0$: Explain
9.- An instructor wishes to see if the way people obtain information is independent of the educational background. A survey of high school and college graduates yielded this information. At $\alpha = 0.01$, test the claim that the way people obtain information is independent of their educational background.

<table>
<thead>
<tr>
<th></th>
<th>T.V</th>
<th>Newspaper</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>High School</td>
<td>70 (66)</td>
<td>65 (66)</td>
<td>62 (66)</td>
</tr>
<tr>
<td>College</td>
<td>30 (34)</td>
<td>35 (34)</td>
<td>38 (34)</td>
</tr>
</tbody>
</table>

$$d.f = (2-1)(3-1) = 1 \cdot 2 = 2$$

$$\chi^2 = \frac{(66-70)^2}{66} + \frac{(66-65)^2}{66} + \frac{(66-62)^2}{66} + \frac{(34-30)^2}{34} + \frac{(34-35)^2}{34} + \frac{(34-38)^2}{34} = 1.47$$

$p$-value: $P = P(\chi^2 > 1.47) = \chi^2_{cdf}(1.47, 1000, 2) = 0.479$

$P = 0.479 > \alpha = 0.01$
10.- True/False section.

(a) For any random sample \( P(\chi^2 < \chi^2_L) = P(\chi^2 > \chi^2_R) \) \( \quad \top \)

(b) For any random sample \( F \geq 1 \) \( \quad \top \)

(c) For any random sample \( \chi^2_L = \chi^2_R \) \( \quad \bot \)

(d) If \( H_a: p < p_0 \) then \( P = P(t < t^*) \) \( \quad \bot \)

(e) For any random sample if \( b < 0 \) then \( r < 0 \) \( \quad \bot \)

(f) For any two random samples \( \hat{p}_1 > \hat{p}_2 \) \( \quad \bot \)

\[ b = \bar{y} - \bar{x} \]